Verifying Array Manipulating Programs by Tiling

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Motivating Example

```
void foo(int A[], int N) {
  for (int i = 0; i < N; i++) {
    if(!(i==0 || i==N-1)) {
      if (A[i] < 5) {
        A[i+1] = A[i] + 1;
        A[i] = A[i-1];
      }
    } else {
      A[i] = 5;
    }
  }
  assert(for k in 0..N-1, A[k]>=5);
}
```

Necessary loop invariant:
$\forall j.(0 \leq j < i) \implies (A[j] \geq 5)$

Goal

- Programs manipulating arrays of parametric size in loops
- Multiple indices updated and/or accessed
- Prove quantified post-conditions over arrays

Classical Hoare logic-based method

- Needs a quantified loop invariant
- Difficult to compute precisely

Verifying Candidate Invariants to prove Post-condition

- Check validity of $\{\forall k.\phi(x) \land B\}$ $\mathsf{L}_{\text{body}}(x, x')$ $\{\forall k.\phi(x')\}$
- $\exists x, x'.\forall k.\phi(x) \land B \land Enc(\mathsf{L}_{\text{body}}(x, x')) \land \exists k.\neg\phi(x')$ must be unsat
- Check validity of $\forall k.\phi(x) \land \neg B \implies \forall k.\psi(x)$
- Both checks are hard as they involve a quantifier alternation

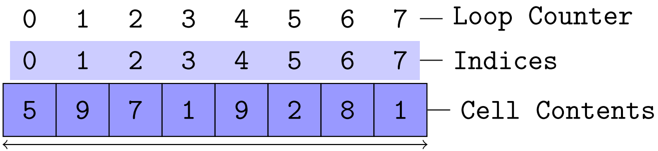
Efficient verification tools available

- Bounded model checkers such as CBMC, Corral
  - Support rich program constructs
  - Do not support quantified reasoning
- SMT solvers such as Z3, CVC4, Yices
  - Support quantified reasoning
  - Can prove small quantified formulas with one or two of alternations
  - Scalability a concern; on-going research

Motivating Example

```
void foo(int A[], int N) {
  for (int i = 0; i < N; i++) {
    if(!(i==0 || i==N-1)) {
      if (A[i] < 5) {
        A[i+1] = A[i] + 1;
        A[i] = A[i-1];
      }
    } else {
      A[i] = 5;
    }
  }
  assert(for k in 0..N-1, A[k]>=5);
}
```

Initial array



$\neg\forall k.a[k] \geq 5$

Motivating Example

```
void foo(int A[], int N) {
  for (int i = 0; i < N; i++) {
    if(!(i==0 || i==N-1)) {
      if (A[i] < 5) {
        A[i+1] = A[i] + 1;
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      }
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    }
  }
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```

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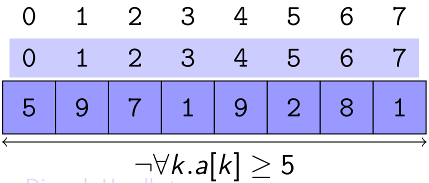

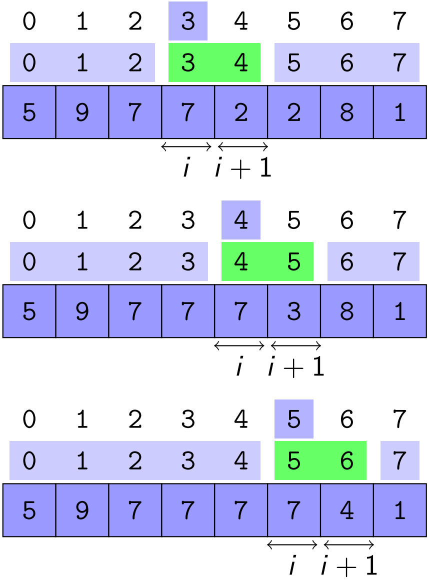

Motivating Example

```
void foo(int A[], int N) {
  for (int i = 0; i < N; i++) {
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        A[i+1] = A[i] + 1;
        A[i] = A[i-1];
      }
    } else {
      A[i] = 5;
    }
  }
  assert(for k in 0..N-1, A[k]>=5);
}
```
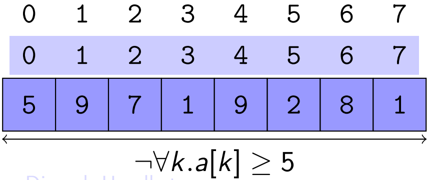
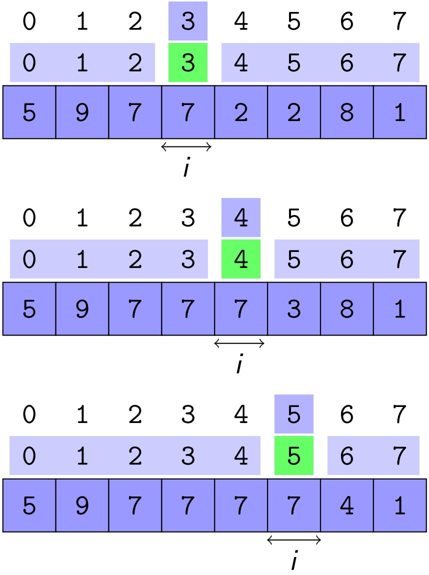

Initial array

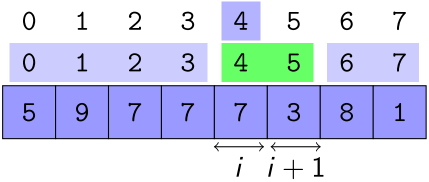




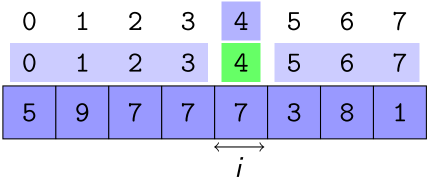
Tiling an Array

A Tile identifies the region of the array where the contribution of a generic loop iteration is localized

Tile : LoopCounter $\times$ Indices $\rightarrow \{\mathbf{tt}, \mathbf{ff}\}$ for loop L

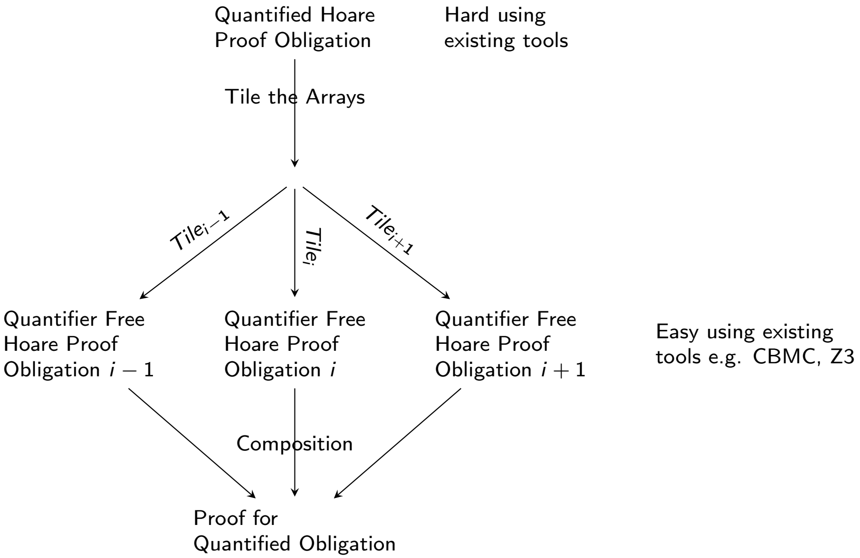


- Tile$(i, j) := i \leq j \leq i + 1$



- Tile$(i, j) := j == i$

Motivation for Tiling



Verification by Tiling

Verify quantified post-conditions over arrays of parametric size

- Programs contain complex array access expressions in loops
- Use candidate quantified invariants
- Use black box back-ends such as SMT solver and BMC's

Inductive Compositional Reasoning

- *Infer* array access patterns in loops
- *Tile* the set of indices using the inferred patterns
- *Slice* the assertion using the tile for a single iteration of the loop
  - Generates quantifier free hoare proof obligations for verification
- Compositionally *prove* universally quantified assertions on arrays
  - Composes verified quantifier free hoare proof obligations

Heuristic Tile Generation

```
void foo(int A[], int N) {
  int j;
  for (int i = 0; i < N; i++) {
    if(!(i==0 || i==N-1)) {
      if (A[i] < 5) {
        A[i+1] = A[i] + 1;
        A[i] = A[i-1];
      }
    } else {
      A[i] = 5;
    }
    if(*) { j=i;   }
    if(*) { j=i+1; }
  }
  assert(for k in 0..N-1, A[k]>=5);
}
```

Using array access patterns

- Store values of updated indices (say in j)
- Use arithmetic invariant generators to infer a relation between i and j
- Infer $\text{Tile}(i,j) := i \leq j \leq i+1$
- Remove overlapping indices $\text{Tile}(i,j) := j == i$

Syntactic Restrictions on Programs

$$
\begin{array}{rcl}
\text{PB} & ::= & \text{St} \\
\text{St} & ::= & v := \text{E} \mid A[\text{E}] := \text{E} \mid \textbf{assume}(\text{BoolE}) \mid \\
 & & \textbf{if}(\text{BoolE})\ \textbf{then}\ \text{St}\ \textbf{else}\ \text{St} \mid \\
 & & \textbf{for}\ (\ell := 0;\ \ell < \text{E};\ \ell := \ell{+}1)\ \{\text{St}\} \mid \\
 & & \text{St} ; \text{St} \\
\text{E} & ::= & \text{E op E} \mid A[\text{E}] \mid v \mid \ell \mid \text{c} \\
\text{BoolE} & ::= & \text{E relop E} \mid \text{BoolE AND BoolE} \mid \\
 & & \text{NOT BoolE} \mid \text{BoolE OR BoolE}
\end{array}
$$

- No unstructured jumps
- Loop counter goes from 0 to some max value
- Assignment statements in body do not update loop counter

Formalization

- Notation
  - I denotes a sequence of array index variables
  - $\mathcal{A}$ is a set of array variables
  - Inv is a (possibly weak) loop invariant for loop L
- Example Post-conditions/assertions
  - $\forall i$ between 0 and N, $A[i]$ is greater equal to minimum
  - $\forall i$ if i is even & between 0 and N then $A[i] = i$
- Formalization of Post-conditions
  - Post $\triangleq \forall I\, (\Phi(I) \implies \Psi(\mathcal{A}, I))$
  - $\Phi(I)$ - quantifier-free formula in theory of arithmetic over integers
  - $\Psi(\mathcal{A}, I)$ - quantifier-free formula in combined theory of arrays and arithmetic over integers

Proving Assertions using Tiles

If following conditions hold on the tile, we have proven the property

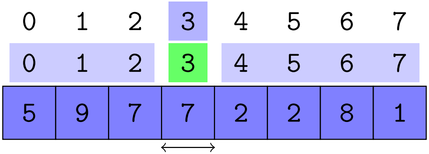
T1: Covers Range relevant to property

T2: Sliced post-condition holds inductively

T3: Non-interference across tiles

T1: Covers Range relevant to Property

Indices of interest must be covered by some *tile*



$$\text{Tile}(i,j) := (j == i)$$

- $\eta_1 \equiv \forall j\,(\Phi(j) \implies \exists i\,(\text{Tile}(i,j)))$
- $\eta_2 \equiv \forall i,j\,(\text{Tile}(i,j) \implies \Phi(j))$
- Validity of $\eta_1 \wedge \eta_2$ ensures T1
- Involves a quantifier alternation; can be handled by SMT solvers

T1: Covers Range relevant to Property

- $\neg(\eta_1 \wedge \eta_2)$ must be unsat
- Negated smt formula is as shown below

```
(declare-fun size () Int)
(declare-fun i () Int)
(declare-fun j () Int)
(assert (or
    (and (>= j 0) (< j size)
        (forall ((i Int))
            (=> (and (>= i 0) (< i size)) (not (= j i)) )))
    (and (>= i 0) (< i size) (= j i)
        (not (and (>= j 0) (< j size))))))
(check-sat)
```

- State-of-the-art solvers can prove unsatisfiability of such formulae

T1: Covers Range relevant to Property

Indices of interest must be covered by some *tile*

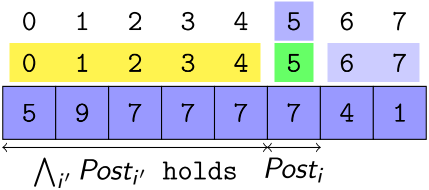
0	1	2	3	4	5	6	7
0	1	2	3	4	5	6	7
5	9	7	7	2	2	8	1

```
void foo(int A[], int N) {
  for (int i = 0; i < N; i++) {
    if(!(i==0 || i==N-1)) {
      if (i%2==0 && A[i] < 5) {
        A[i+2] = A[i] + 1;
        A[i] = A[i-2];
      }
    } else {
      A[i] = 5;
    }
  }
  assert(for k in 0..N-1 && k%2==0, A[k]>=5);
}
```

- Tiles need not cover full-range of indices
- Non-compact tiles are allowed
- Tiles cover range relevant to the property

T2: Sliced Post-condition holds Inductively

Post-condition wrt indices in the i^{th} tile holds inductively



- Sliced post-condition for the i^{th} tile
  $\mathsf{Post}_i \triangleq \forall j\,(\mathsf{Tile}(i,j) \wedge \Phi(j) \implies \Psi(\mathsf{A},j))$

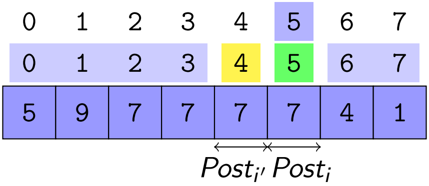
- $\{\mathsf{Inv} \wedge \bigwedge_{i':0\le i'<i} \mathsf{Post}_{i'}\}\ \mathrm{L_{body}}\ \{\mathsf{Inv} \wedge \mathsf{Post}_i\}$ must be valid

- Post_i and $\bigwedge_{i':0\le i'<i} \mathsf{Post}_{i'}$ are universally quantified formula

- Hoare logic-based reasoning tools that permit quantification have limited automation and scalability

T2*: Sliced Post-condition holds Inductively

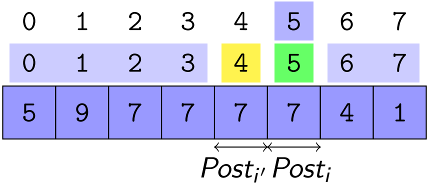
Post-condition wrt indices in the i^{th} tile holds inductively



- $\mathsf{L}_{\mathrm{body}}$ is a loop free fragment of the code
- $Rd_{\mathsf{L}}(i) = \{i, i-1\}$ - Finite set of indices read in i^{th} iteration
- $\zeta(i) := \bigwedge_{e_k \in Rd_{\mathsf{L}}(i)} \big(((0 \leq i' < i) \wedge \mathsf{Tile}(i', e_k) \wedge \Phi(e_k)) \Rightarrow \Psi(\mathsf{A}, e_k) \big)$
- $\{\mathsf{Inv} \wedge \zeta(i) \wedge \mathsf{Tile}(i,j) \wedge \Phi(j)\} \mathsf{L}_{\mathrm{body}} \{\mathsf{Inv} \wedge \Psi(\mathsf{A}, j)\}$ must be valid

T2*: Sliced Post-condition holds Inductively

Post-condition wrt indices in the i^{th} tile holds inductively



T2 - $\{\mathsf{Inv} \wedge \bigwedge_{i':0\leq i'<i} \mathsf{Post}_{i'}\}\ \mathsf{L}_{\text{body}}\ \{\mathsf{Inv} \wedge \mathsf{Post}_i\}$

T2* - $\{\mathsf{Inv} \wedge \zeta(i) \wedge \mathsf{Tile}(i,j) \wedge \Phi(j)\}\mathsf{L}_{\text{body}}\{\mathsf{Inv} \wedge \Psi(\mathsf{A},j)\}$

- ⋆ T2* $\Rightarrow$ T2 and T2 $\not\Rightarrow$ T2*
- ⋆ T2* is now a quantifier free formula and can be checked using bmc

T2*: Sliced Post-condition holds Inductively

Original Program

```
void foo(int A[], int N) {
  for (int i = 0; i < N; i++)
  {

    if(!(i==0 || i==N-1)) {
      if (A[i] < 5) {
        A[i+1] = A[i] + 1;
        A[i] = A[i-1];
      }
    } else {
      A[i] = 5;
    }

  }
  assert(for k in 0..N-1, A[k]>=5);
}
```

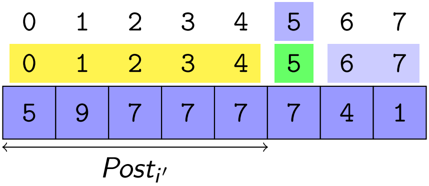
Transformed Program

```
i=*;  j=*;  jp=*;
assume(0 <= i < N);
assume(j == i);
assume(jp == i-1);
assume(A[jp] >= 5);

if(!(i==0 || i==N-1)) {
  if (A[i] < 5) {
    A[i+1] = A[i] + 1;
    A[i] = A[i-1];
  }
} else {
  A[i] = 5;
}

assert(A[j] >= 5);
```

- Use CBMC to ensure T2* by checking the loop free code

T3: Non-interference(property) across Tiles

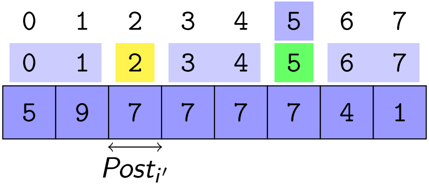
No iteration $i > i'$ interferes with the truth of $Post_{i'}$, once established



- Sliced post-condition for the i'^{th} tile
  $\mathrm{Post}_{i'} \triangleq \forall j' \, (\mathrm{Tile}(i', j') \wedge \Phi(j') \implies \Psi(\mathrm{A}, j'))$
- $\{\mathrm{Inv} \wedge \bigwedge_{i':0 \leq i' < i} \mathrm{Post}_{i'}\} \mathrm{L}_{\mathrm{body}} \{\bigwedge_{i':0 \leq i' < i} \mathrm{Post}_{i'}\}$ must be valid
- Post_i and $\bigwedge_{i':0 \leq i' < i} \mathrm{Post}_{i'}$ are universally quantified formula
- Hoare logic-based reasoning tools that permit quantification have limited automation and scalability

T3*: Non-interference(property) across Tiles

No iteration $i > i'$ interferes with the truth of $Post_{i'}$, once established



T3 - $\{\mathsf{Inv} \wedge \bigwedge_{i':0 \leq i' < i} \mathsf{Post}_{i'}\} \mathsf{L}_{\text{body}} \{\bigwedge_{i':0 \leq i' < i} \mathsf{Post}_{i'}\}$

T3* - $\{\mathsf{Inv} \wedge (0 \leq i' < i) \wedge \mathsf{Tile}(i', j') \wedge \Phi(j') \wedge \Psi(\mathsf{A}, j')\}\ \mathsf{L}_{\text{body}}\ \{\Psi(\mathsf{A}, j')\}$

i' and j' be free variables here

- ⋆ T3* ⇒ T3 and T3 ⇒ T3*
- ⋆ T3* is now a quantifier free formula and can be checked using bmc

T3*: Non-interference(property) across Tiles

Original Program

```
void foo(int A[], int N) {
  for (int i = 0; i < N; i++)
  {

    if(!(i==0 || i==N-1)) {
      if (A[i] < 5) {
        A[i+1] = A[i] + 1;
        A[i] = A[i-1];
      }
    } else {
      A[i] = 5;
    }

  }
  assert(for k in 0..N-1, A[k]>=5);
}
```

Transformed Program

```
i=*;  ip=*;  jp=*;
assume(0 <= i < N);
assume(0 <= ip < i);
assume(jp == ip);
assume(A[jp] >= 5);

if(!(i==0 || i==N-1)) {
  if (A[i] < 5) {
    A[i+1] = A[i] + 1;
    A[i] = A[i-1];
  }
} else {
  A[i] = 5;
}

assert(A[jp] >= 5);
```

- Use CBMC to ensure T3* by checking the loop free code

Inductive Compositional Reasoning

- Inductive Reasoning

  T2 Sliced post-condition holds for each iteration

- Compositional Reasoning

  T3 Truth of sliced post-condition once established is not altered subsequently

  T1 Tiles cover the entire range of array indices of interest

Theorem

Suppose $\mathsf{Tile} : \mathsf{LoopCounter} \times \mathsf{Indices} \rightarrow \{\mathbf{tt}, \mathbf{ff}\}$ *satisfies T1, T2 and T3. If* $\mathsf{Pre} \Rightarrow \mathsf{Inv}$ *holds and the loop* L *iterates at least once, then the Hoare triple* $\{\mathsf{Pre}\}\ \mathsf{L}\ \{\mathsf{Post}\}$ *holds.*

Proof.

The proof proceeds by induction on values of LoopCounter (say i).

Given:

$$\text{Pre} \Rightarrow \text{Inv} \tag{1}$$

Base Case:

Prove $\{\text{Pre}\}\ \ \text{L}_{\text{body}}\ \ \{\text{Post}_i\}$ holds, where $i = 0$

$$\{\text{Inv}\}\ \ \text{L}_{\text{body}}\ \ \{\text{Inv} \wedge \text{Post}_i\} \qquad (\because T2) \tag{2}$$

$$\{\text{Pre}\}\ \ \text{L}_{\text{body}}\ \ \{\text{Inv} \wedge \text{Post}_i\} \qquad (\because \textit{From } (1)\ \&\ (2)) \tag{3}$$

$$\{\text{Pre}\}\ \ \text{L}_{\text{body}}\ \ \{\text{Post}_i\} \qquad (\because \text{Inv} \wedge \{\text{Post}_i\} \Rightarrow \{\text{Post}_i\}) \tag{4}$$

Induction Hypothesis:

$$\{\text{Pre}\}\ \ (\text{L}_{\text{body}})^{i'}\ \ \{\bigwedge_{i':0 \leq i' < i} \text{Post}_{i'}\} \qquad (\because T3) \tag{5}$$

Proof. Cont...

Induction:

Assuming hypothesis, prove $\{\mathsf{Pre}\}\ \ (\mathsf{L}_{\text{body}})^{i'}\ \ \{\bigwedge_{i':0\leq i'\leq i} \mathsf{Post}_{i'}\}$ holds.

$$\{\mathsf{Inv} \wedge \bigwedge_{i':0\leq i'<i} \mathsf{Post}_{i'}\}\ \ \mathsf{L}_{\text{body}}\ \ \{\mathsf{Inv} \wedge \mathsf{Post}_i\} \qquad (\because T2) \quad (6)$$

$$\{\mathsf{Inv} \wedge \bigwedge_{i':0\leq i'<i} \mathsf{Post}_{i'}\}\ \ \mathsf{L}_{\text{body}}\ \ \{\mathsf{Post}_i\} \qquad (\because \mathsf{Inv} \wedge \mathsf{Post}_i \Rightarrow \mathsf{Post}_i) \quad (7)$$

At the end of the i^{th} iteration of the loop L the following Hoare triple holds:

$$\{\mathsf{Pre}\}\ \ (\mathsf{L}_{\text{body}})^{i'}\ \ \{\bigwedge_{i':0\leq i'\leq i} \mathsf{Post}_{i'}\} \qquad (\because \textit{From } (5)\ \&\ (7)) \qquad (8)$$

$$\bigwedge_{i} \mathsf{Post}_i \equiv \mathsf{Post} \qquad (\because T1) \qquad (9)$$

$$\{\mathsf{Pre}\}\ \ \mathsf{L}\ \ \{\mathsf{Post}\} \qquad (\because \textit{From } (8)\ \&\ (9)) \qquad \square$$

Sequentially Composed Loops

```
void copynswap(int N)
{
 int i, tmp;
 int a[N], b[N], acopy[N];

 for (i = 0; i < N; i++) {
   acopy[i] = a[i];
 }

 for (i = 0; i < N; i++) {
   tmp = a[i];
   a[i] = b[i];
   b[i] = tmp;
 }

 for (i = 0; i < N; i++) {
   assert(b[i] == acopy[i]);
 }
}
```

Mid-conditions

- Invariants between sequentially composed loops
- Hard to generate precise invariants
- Identify *candidate* mid-conditions using annotation assistants
- *Prove* them using Tiling

Candidate mid-conditions

- $\forall i(a[i] = acopy[i])$
- $\forall i(a[i] \neq b[i])$

Proved mid-conditions

- $\forall i(a[i] = acopy[i])$

Nested Loops

```
void nested(int N)
{
  int i, j, VAL=2, arr[N];

  assume(N % 5 == 0);
  for(i = 1; i <= N/5; i++)
  {
    for(j = 1; j <= 5; j++)
    {
      if(j >= VAL)
        arr[i*5 - j] = j;
      else
        arr[i*5 - j] = 0;
    }
  }

  assert(for i in 0..N-1 (arr[i]>=VAL || arr[i]==0));
}
```

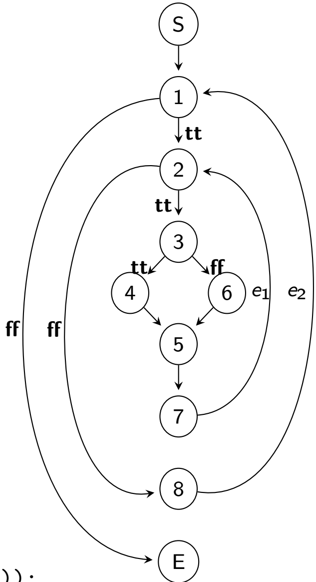


Figure: CFG

CFG, Cutpoints and Segments

Back-edge - Edge from a node within the body of a loop to the node representing the corresponding loop head

- Edges e_1 and e_2

Cut-Points - Target nodes of back-edges and Start and End nodes

- Nodes 1, 2, S, E

Segments - Acyclic sub-graph of a CFG

- starts from a cut-point and ends at another cut-point
- does not pass through any other cut-point in between

$S \mapsto 1$, $1 \mapsto 2$,
$2 \mapsto 3 \mapsto \{4, 6\} \mapsto 5 \mapsto 7 \mapsto 2$,
$2 \mapsto 8 \mapsto 1$, $1 \mapsto E$ are segments

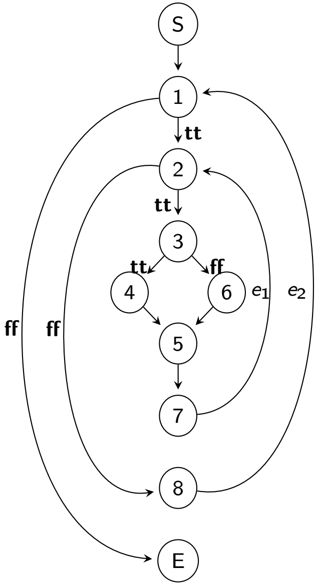


Figure: CFG

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Verifying Nested Loops using Segments

Analysis now applies to each segment in the topological order

- c_1 and c_2 be two cut-points
- Segment s starts at c_1, ends at c_2
- Inv_{c_1} and Inv_{c_2} are candidate invariants at c_1 and c_2
- Generate tiles for each segment
- Check T1, T2*, T3* for each segment s
- Invariants for the nested loop example
  - $Inv_1 := \forall k(0 \leq k < 5 * i) \implies (arr[k] >= VAL || arr[k] == 0)$
  - $Inv_2 := \forall k(5*i-j \leq k < 5*i) \implies (arr[k] >= VAL || arr[k] == 0)$

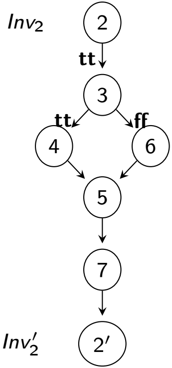

Figure: Verifying a Segment

Tiler Tool Diagram & Artifact Details

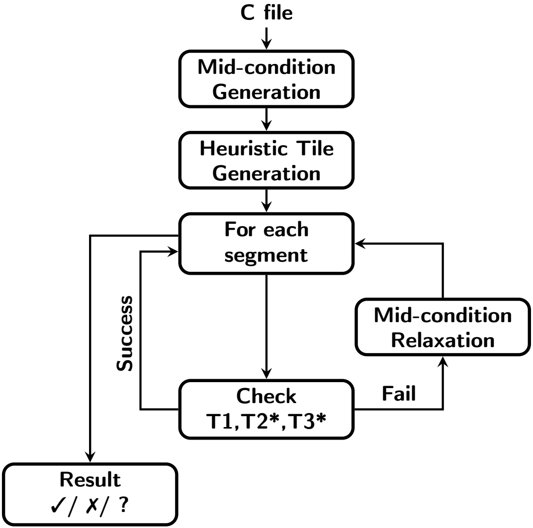

Figure: Tiler Tool Diagram

Tiler Artifact

- VM image of Ubuntu 16.04 LTS
- Contains
  - Source code
  - Build files
  - Benchmarks

Tiler Implementation

- Built on top of LLVM/CLANG infrastructure in C++
- Mid-condition generation
  - Daikon learns candidate scalar invariants from concrete traces
  - Lift these to quantified invariants
- Heuristic tile generation
  - Determine indices in terms of loop counters
  - Get a closed form expression in terms of index expressions
  - Remove possible overlaps
- Checking conditions T1, T2 and T3
  - Z3 for checking the validity of T1
  - CBMC for checking the validity of T2 and T3

Tiler Benchmarking

- 60 benchmarks from industry and academia
- Performance compared with tools
  - SMACK+Corral - Bounded model checker
  - Booster - Acceleration based verification for arrays
  - Vaphor - Distinguished cell abstraction for arrays
- Memory limit - 1GB
- Time limit - 900s

Tiler in Action

Benchmark	#L	**Tiler**	S+C	Booster	Vaphor
cpynrev.c	2	✓3.8	†	✓3.1	✓5.4
cpynswp.c	2	✓4.2	†	✓12.4	✓1.38
cpynswp2.c	3	✓10.2	†	✓198	✓7.2*
maxinarr.c	1	✓0.51	†	✓0.01	✓0.11
mininarr.c	1	✓0.53	†	✓0.02	✓0.13
poly1.c	1	TO	†	✓15.7	TO
poly2.c	2	**?** 6.44	†	**?** 19.5	TO
tcpy.c	1	**?** 0.65	†	TO	✓25.1
rew.c	1	✓0.48	†	✓0.01	TO
skipped.c	1	✓1.24	†	TO	TO
rewrev.c	1	✓0.39	†	TO	TO
pr4.c	1	✓0.68	†	TO	TO
pr5.c	1	✓1.32	†	TO	TO
pnr4.c	1	✓0.86	†	TO	TO
pnr5.c	1	✓1.98	†	TO	TO
mbpr4.c	4	✓12.75	†	TO	TO
mbpr5.c	5	✓18.08	†	TO	TO
nr4.c	1-1	✓2.43*	†	TO	TO
nr5.c	1-1	✓2.90*	†	TO	TO
copy9u.c	9	✗0.16	✗4.48	✗0.44	✗30.8
skippedu.c	1	✗0.81	✗2.94	✗0.02	TO

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Battery Voltage Regulator

```
void BVR(int N, int MIN)
{
  int i, volArray[N];

  if(N % 4 != 0) { return; }

  for(i = 1; i <= N/4; i++)
  {
    if(1 >= MIN)
      volArray[i*4-1] = 1;
    else
      volArray[i*4-1] = 0;
    if(3 >= MIN)
      volArray[i*4-2] = 3;
    else
      volArray[i*4-2] = 0;
    if(7 >= MIN)
      volArray[i*4-3] = 7;
    else
      volArray[i*4-3] = 0;
    if(5 >= MIN)
      volArray[i*4-4] = 5;
    else
      volArray[i*4-4] = 0;
  }

  for(i = 0; i < N; i++)
  {
    assert(volArray[i] >= MIN ||
      volArray[i] == 0);
  }
}
```

$$\text{Tile}(i,j) := 4*i-4 \leq j < 4*i$$

Skipped Indices

```
void skip(int N)
{
  int i;
  int a[N];

  if(N % 2 != 0)
  {
    return;
  }

  assume(N % 2 == 0);
  for(i = 1; i <= N/2; i++ )
  {
    if( a[2*i-2] > 2*i-2 )
    {
      a[2*i-2] = 2*i-2;
    }

    if( a[2*i-1] > 2*i-1 )
    {
      a[2*i-1] = 2*i-1;
    }
  }

  for(i = 0; i < N; i++)
  {
    assert(a[i] <= i);
  }
  return;
}
```

$\text{Tile}(i, j) := 2 * i - 2 \leq j < 2 * i$

Array Reversal

```
void revcopynswap(int N)
{
  int i;
  int tmp;
  int a[N];
  int b[N];
  int rev_copy[N];

  for(i = 0; i < N; i++)
  {
    rev_copy[N-i-1] = a[i];
  }

  for(i = 0; i < N; i++)
  {
    tmp = a[i];
    a[i] = b[i];
    b[i] = tmp;
  }

  for(i = 0; i < N; i++)
  {
    assert(b[i] == rev_copy[N-i-1]);
  }
}
```

Loop 1 - $\text{Tile}(i, j) := j == N - i - 1$

Loop 2 - $\text{Tile}(i, j) := j == i$

Tiles in Benchmarks

- **Reverse** the contents of the array
  - $\text{Tile}(i,j) := j == N - i - 1$
- A **bunch** of indices updated in a loop
  - $\text{Tile}(i,j) := 2 * i - 2 \leq j < 2 * i$
  - $\text{Tile}(i,j) := 3 * i - 3 \leq j < 3 * i$
  - $\text{Tile}(i,j) := 4 * i - 4 \leq j < 4 * i$
- **Adjacent** indices to the counter
  - $\text{Tile}(i,j) := j == i - 1$
  - $\text{Tile}(i,j) := j == i + 1$
- Most **common** tile in array processing loops
  - $\text{Tile}(i,j) := j == i$

Limitations

```
void tcpy(int N)
{
  int i, a[N], reverse[N];

  if(N % 2 != 0)
  { return; }

  assume(N % 2 == 0);
  for (i = 0; i < N/2; i++)
  {
     reverse[i] = a[N-i-1];
     reverse[N-i-1] = a[i];
  }

  for(i = 0; i < N; i++)
  {
     assert(a[i] = reverse[N-i-1]);
  }
}
```

```
void poly2(int N)
{
  int i, a[N];

  for(i=0; i<N; i++)
  {
    a[i] = i*i + 2;
  }

  for(i=0; i<N; i++)
  {
    a[i] = a[i] - 2;
  }

  for(i=0; i<N; i++)
  {
    assert(a[i] == i*i);
  }
}
```

Related Work

- Abstract Interpretation based
  - Jiangchao Liu and Xavier Rival (2015). "Abstraction of Arrays Based on Non Contiguous Partitions". In: *VMCAI'15*, pp. 282–299
  - Patrick Cousot, Radhia Cousot, and Francesco Logozzo (2011). "A parametric segmentation functor for fully automatic and scalable array content analysis". In: *POPL'11*, pp. 105–118
  - Sumit Gulwani, Bill McCloskey, and Ashish Tiwari (2008). "Lifting abstract interpreters to quantified logical domains". In: *POPL'08*, pp. 235–246
- Abstraction based
  - David Monniaux and Laure Gonnord (2016). "Cell Morphing: From Array Programs to Array-Free Horn Clauses". In: *SAS'16*, pp. 361–382
  - Francesco Alberti, Silvio Ghilardi, and Natasha Sharygina (2014). "Booster: An Acceleration-Based Verification Framework for Array Programs". In: *ATVA'14*, pp. 18–23
- Without explicit partitioning
  - Isil Dillig, Thomas Dillig, and Alex Aiken (2010). "Fluid Updates: Beyond Strong vs. Weak Updates". In: *ESOP'10*, pp. 246–266

Conclusion and Future Work

- Presented a novel verification technique that
  - proves universally quantified assertions over arrays
  - decomposes reasoning about arrays using *tiles*
  - is property driven, compositional and efficient
- Future directions
  - Automated synthesis of *tiles*
  - Combining the strengths of Booster, Vaphor and Tiler
  - Integration of other candidate invariant generators

Thank you