

Diffy: Inductive Reasoning of Array Programs using Difference Invariants



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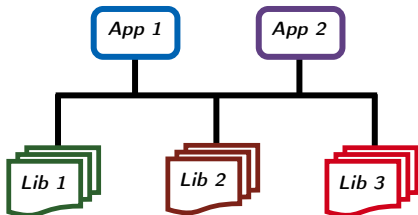
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Motivation



Photo by Sahin Sezer Dincer from Pexels



- Software in safety critical applications often use arrays
- Software developed with parametric array sizes
 - ▶ Deployment-specific parameter instantiation
- Array libraries with parametric array sizes used across applications
- Verifying parametric properties of such software important

Program with Parametric Array Size

assume($\forall x \in [0, N), A[x] = N$)

$\varphi(N)$

```
1. void foo(int A[], int N)
2. {
3.   int S = 0;
4.   for(int i=0; i<N; i++)
5.     for(int j=0; j<N; j++)
6.       if(i+j < N)
7.         S = S + A[i+j];
8.   for(int k=0; k<N; k++)
9.     for(int l=0; l<N; l++)
10.      A[l] = A[l] + 1;
11.   A[k] = A[k] + S;
12. }
```

Arrays of parametric size N

Nested loops and branch conditions dependent on N

P_N Quantified (possibly non-linear) pre- and post-conditions

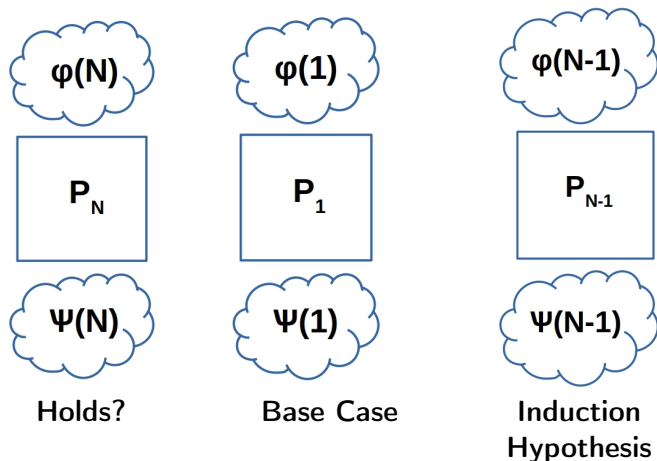
Prove the parametric Hoare triple $\{\varphi(N)\} P_N \{\psi(N)\}$ for all $N > 0$

assert($\forall x \in [0, N), A[x] = N*(N+5)/2$) $\psi(N)$

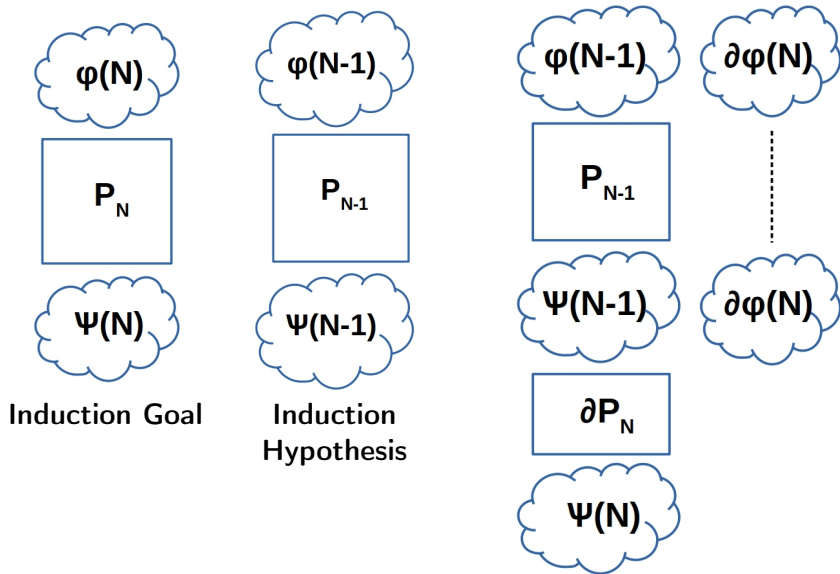
Earlier Work on Proving Programs with Arrays

- ★ Alberti et. al.'2014 - Lazy abstraction based interpolation and acceleration
- ★ Monniaux, Gonnord'2016 - Abstraction to array-free Horn formulas
- ★ Chakraborty et. al.'2017 - Relate array indices and loop counters for inductive proofs
- ★ Fedyukovich et. al.'2019 - Infer quantified invariants in CHCs
- ★ Rajkhowa & Lin'2019 - Inductive encoding of arrays as uninterpreted functions
- ★ Afzal et. al.'2020 - Output abstraction, Loop shrinking/pruning
- ★ Chakraborty et. al.'2020 - Full-Program Induction (FPI)

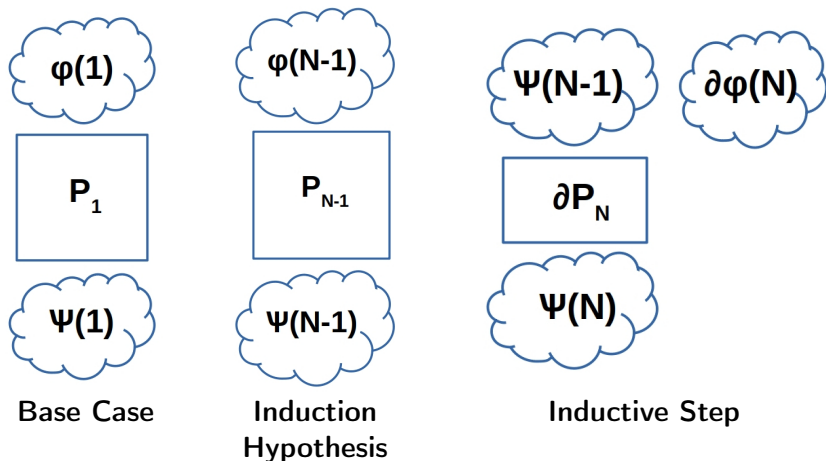
Full-Program Induction (FPI)



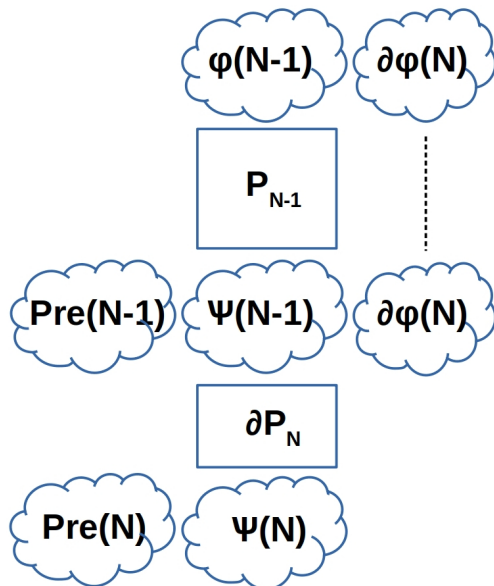
Full-Program Induction (FPI)



Full-Program Induction (FPI)



Full-Program Induction (FPI)



Pros of Full-Program Induction

- Simplifies verification when ∂P_N is loop-free
- Computing P_{N-1} , $\psi(N-1)$ requires a simple substitution
- $\psi(N-1)$ directly useful during the inductive step
- Computing ∂P_N is simple for certain classes of programs
 - ▶ Programs with no data dependence across loops and loop iterations
 - ▶ No variable or array element's value depends on N
 - ▶ "Peeled" iterations of loops suffices as ∂P_N

∂P_N can be computed in other cases too, but ...

Difficulties with Full-Program Induction

```
x = 0;
for(i=0; i<N; i++)
  x = x + N*N;
  a[i] = a[i] + N;
```

```
for(j=0; j<N; j++)
  b[j] = x + j;
```

P_N

Constructing ∂P_N can get non-trivial in some cases

Computed ∂P_N may have loops

- ▶ Two loops in this example

Analyzing ∂P_N as hard as P_N

Recursive application of FPI on ∂P_N may be required **even for non-nested loops**

```
x = 0;
for(i=0; i<N-1; i++)
  x = x+(N-1)*(N-1);
  a[i] = a[i] + N-1;
for(j=0; j<N-1; j++)
  b[j] = x + j;
-----
for(i=0; i<N-1; i++)
  x = x + 2*N-1;
  a[i] = a[i] + 1;
x = x + N*N;
a[N-1] = a[N-1]+N;
for(k=0; k<N-1; k++)
  b[k] = b[k] +
  (N-1)*(2*N-1)+N*N;
b[N-1] = x + N-1;
```

P_{N-1}

∂P_N

Additional Difficulties with FPI - Branch Conditions

$\{ \varphi(N) \} P_N \{ \psi(N) \}$

`assume(true)`

```
1. void foo(int A[], int N) {
2.   for (int i=0; i<N; i++)
3.     A[i] = 0;
4.   for (int j=0; j<N; j++) {
5.     if(N%2 == 0) {
6.       A[j] = A[j] + 2;
7.     } else {
8.       A[j] = A[j] + 1;
9.     }
10.  }
11.}
```

`assert($\forall x \in [0, N)$, A[x] = N%2)`

$\{ \varphi(N-1) \} P_{N-1} \{ \psi(N-1) \}$

`assume(true)`

```
1. void foo(int A[], int N) {
2.   for (int i=0; i<N-1; i++)
3.     A[i] = 0;
4.   for (int j=0; j<N-1; j++) {
5.     if((N-1)%2 == 0) {
6.       A[j] = A[j] + 2;
7.     } else {
8.       A[j] = A[j] + 1;
9.     }
10.  }
11.}
```

`assert($\forall x \in [0, N-1)$, A[x] = (N-1)%2)`

Additional Difficulties with FPI - Nested Loops

$\{ \varphi(N) \} P_N \{ \psi(N) \}$

assume(true)

```
1. void foo(int A[], int N) {
2.   int S=0;
3.   for(int i=0; i<N; i++) A[i] = 0;
4.   for(int j=0; j<N; j++) S = S + 1;
5.   for(int k=0; k<N; k++) {
6.     for(int l=0; l<N; l++) {
7.       A[l] = A[l] + 1;
8.     }
9.     A[k] = A[k] + S;
10.  }
11.}
```

assert($\forall x \in [0, N), A[x] = 2 * N$)

$\{ \varphi(N - 1) \} P_{N-1} \{ \psi(N - 1) \}$

assume(true)

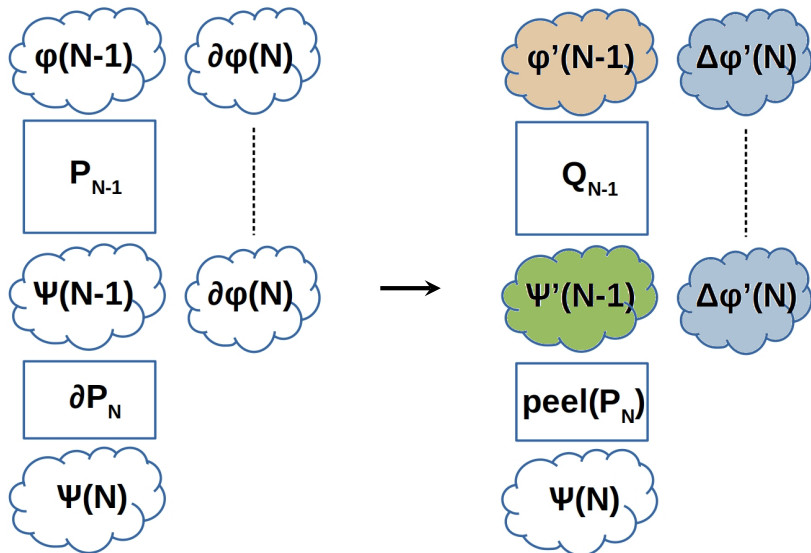
```
1. void foo(int A[], int N) {
2.   int S=0;
3.   for(int i=0; i<N-1; i++) A[i] = 0;
4.   for(int j=0; j<N-1; j++) S = S + 1;
5.   for(int k=0; k<N-1; k++) {
6.     for(int l=0; l<N-1; l++) {
7.       A[l] = A[l] + 1;
8.     }
9.     A[k] = A[k] + S;
10.  }
11.}
```

assert($\forall x \in [0, N-1), A[x] = 2 * (N-1)$)

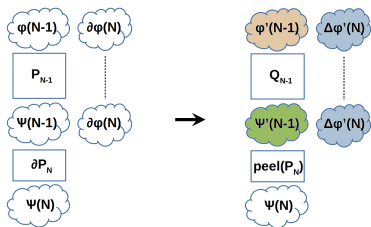
A Relook at Full-Program Induction

- Idea of inducting over full-program interesting
- Allows easy access and use of the induction hypothesis
- Can we somehow reduce the difficulty of generating of ∂P_N ?
 - ▶ Caveat: No free lunches
- There is a trade-off
 - ★ Ease of generating difference code
 - vs
 - ★ Ease of using induction hypothesis

Utilizing the Available Trade-off



Utilizing the Available Trade-off



- $\text{peel}(P_N)$ is just the peeled iterations of loops
- Q_{N-1} differs from P_N only in loop bounds
- $\varphi'(N-1)$, $\Delta\varphi'(N)$ similar to (not always same as) $\varphi(N-1)$, $\Delta\varphi(N)$
- $\psi'(N-1)$ derived from $\psi(N-1)$ and “differences” b/w P_{N-1} , Q_{N-1}

Ensure $\{\varphi(N-1)\} P_{N-1} \{\psi(N-1)\} \Rightarrow \{\varphi'(N-1)\} Q_{N-1} \{\psi'(N-1)\}$

Overcoming Difficulties with FPI

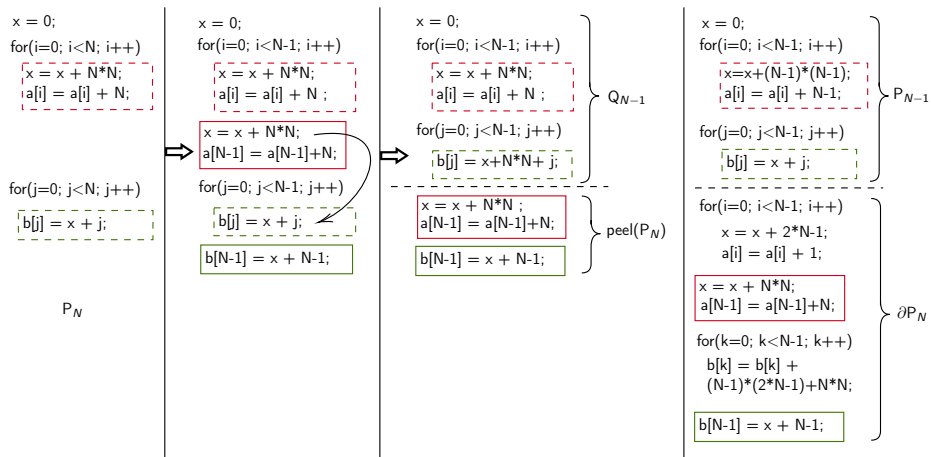
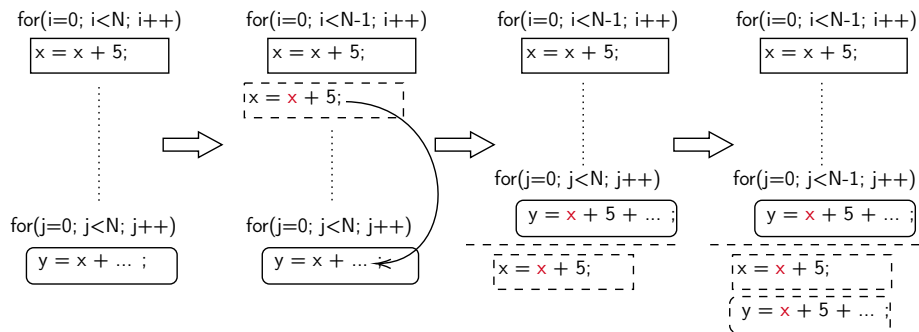
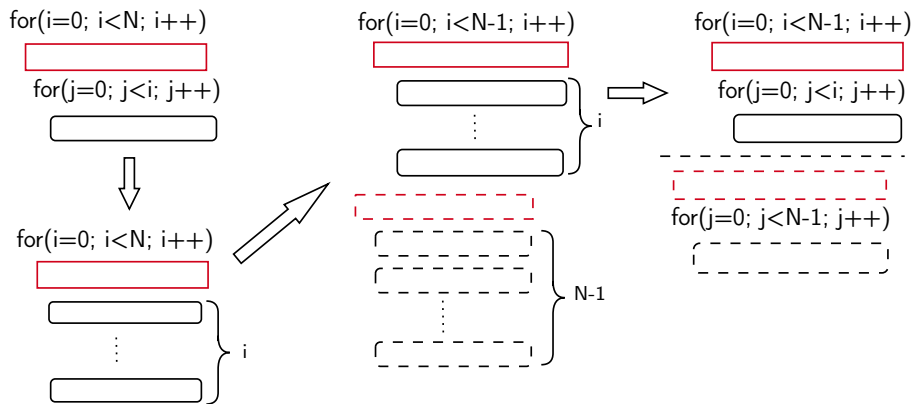


Figure: Algorithmic Construction of Programs Q_{N-1} and $\text{peel}(P_N)$

Computing Q_{N-1} and peel(P_N)



Computing Q_{N-1} and peel(P_N)

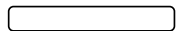


Computing Q_{N-1} and peel(P_N)

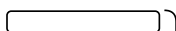
for(i=0; i<N; i++)



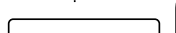
for(j=0; j<N; j++)



for(i=0; i<N; i++)



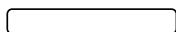
...



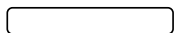
} N



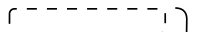
for(i=0; i<N-1; i++)



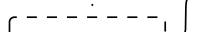
...



} N-1



...



} N-1



...

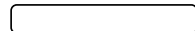


} N

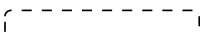
for(i=0; i<N-1; i++)



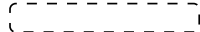
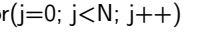
for(j=0; j<N-1; j++)



for(i=0; i<N-1; i++)



for(j=0; j<N; j++)



Syntactic Restrictions on Programs

$PB ::= St$
 $St ::= v := E \mid A[E] := E \mid St; St \mid \text{if}(\text{BoolE}) \text{ then } St \text{ else } St \mid$
 $\text{for } (l := 0; l < UB; l := l+1) \{St\}$
 $UB ::= UB \text{ op } UB \mid l \mid c \mid N$
 $E ::= E \text{ op } E \mid A[E] \mid v \mid l \mid c \mid N$
 $\text{op} ::= + \mid - \mid * \mid /$
 $\text{BoolE} ::= E \text{ relop } E \mid \text{BoolE AND BoolE} \mid \text{NOT BoolE} \mid$
 BoolE OR BoolE

- No unstructured jumps
- Assignment statements in body do not update loop counter
- UB expressions is in terms of N and outer loop counters
- Nested loop is the last loop in the sequence

Correctness and Progress Guarantees

Lemma

Suppose

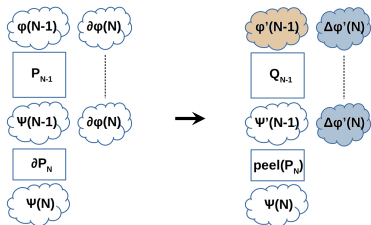
- 1) *Program P_N satisfies our syntactic restrictions*
- 2) *Upper bound expressions of all loops are linear expressions in N , the loop counters of outer loops*

Max loop nesting depth in $\text{peel}(P_N) < \text{Max loop nesting depth in } P_N$

Theorem

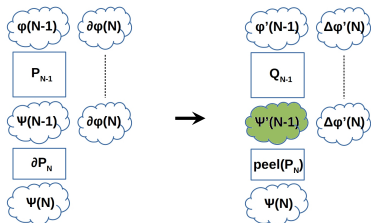
$$Q_{N-1}; \text{peel}(P_N) \equiv P_N$$

Computing $\varphi'(N-1)$ and $\Delta\varphi'(N)$



- Suppose $\varphi(N) \equiv \forall i \in [0, N), a[i] = N$
- $\nexists \partial\varphi(N)$ s.t. $\varphi(N) \Rightarrow \varphi(N-1) \wedge \partial\varphi(N)$
- $\exists \Delta\varphi'(N), \varphi'(N-1)$ s.t. $\varphi(N) \Rightarrow \varphi'(N-1) \wedge \Delta\varphi'(N)$
 - ▶ $\varphi'(N-1) \equiv \forall i \in [0, N-1), a[i] = N$
 - ▶ $\Delta\varphi'(N) \equiv a[N-1] = N$
- Q_{N-1} must not modify arrays/variables in $\Delta\varphi'(N)$

Computing $\psi'(N-1)$



- Infer relations between variables in Q_{N-1} , P_{N-1}
 - ▶ **Difference invariant:** $D(V_Q, V_P)$
- $\psi'(N-1) \equiv (\exists V_P. (\psi(N-1) \wedge D(V_Q, V_P)))$
- Finding sufficient $D(V_Q, V_P)$ is often “**simpler**” than generating invariants to prove the post-condition directly
- Guess-and-check $D(V_Q, V_P)$ using simple templates
 - ▶ $\bullet = \bullet + f_1(N)$ and $\forall i. \bullet [g(i)] = \bullet [g(i)] + f_2(i, N)$
 - ▶ f, g linear/quadratic

Computing $\psi'(N - 1)$

Loop invariants:

$$\phi(N) \equiv \forall i \in [0, N), a[i] = N$$

```
void foo(int a[], int N)
{
  int b[N], x = 0;

  for(int i=0; i<N; i++) {
    x = x + a[i];
  }

  for(int j=0; j<N; j++) {
    b[j] = x + j;
  }
}
```

P_N

$$x = i \times N$$

$$\forall k_1 \in [0, j), b[k_1] = k_1 + N^2$$

$$\psi(N) \equiv \forall i \in [0, N), b[i] = i + N^2$$

Computing $\psi'(N-1)$

Difference invariants $D(V_Q, V_P)$:

$$\phi(N-1) \equiv \forall i \in [0, N-1), a[i] = N-1$$

$$\forall i \in [0, N-1), a'[i] - a[i] = 1$$

$$\phi'(N-1) \equiv \forall i \in [0, N-1), a'[i] = N$$

```
void foo(int a[], int N)
```

```
{  
  int b[N-1], x = 0;  
  
  for(int i=0; i<N-1; i++) {  
    x = x + a[i];  
  }
```

```
  for(int j=0; j<N-1; j++) {  
    b[j] = x + j;  
  }
```

```
}
```

P_{N-1}

```
void bar(int a[], int N)
```

```
{  
  int b'[N-1], x' = 0;  
  
  for(int i=0; i<N-1; i++) {  
    x' = x' + a'[i];  
  }
```

```
  for(int j=0; j<N-1; j++) {  
    b'[j] = x' + N + j;  
  }
```

```
}
```

Q_{N-1}

$$\psi'(N-1) \equiv \exists V_P. (\psi(N-1) \wedge D(V_Q, V_P))$$

$$\psi(N-1) \equiv \forall i \in [0, N-1), b[i] = i + (N-1)^2$$

$$\psi'(N-1) \equiv ??$$

Computing $\psi'(N-1)$

Difference invariants $D(V_Q, V_P)$:

$$\phi(N-1) \equiv \forall i \in [0, N-1], a[i] = N-1$$

$$\forall i \in [0, N-1], a'[i] - a[i] = 1$$

$$\phi'(N-1) \equiv \forall i \in [0, N-1], a'[i] = N$$

```
void foo(int a[], int N)
{
  int b[N-1], x = 0;

  for(int i=0; i<N-1; i++) {
    x = x + a[i];
  }

  for(int j=0; j<N-1; j++) {
    b[j] = x + j;
  }
}
```

P_{N-1}

```
void bar(int a'[], int N)
{
  int b'[N-1], x' = 0;

  for(int i=0; i<N-1; i++) {
    x' = x' + a'[i];
  }

  for(int j=0; j<N-1; j++) {
    b'[j] = x' + N + j;
  }
}
```

Q_{N-1}

$$\psi'(N-1) \equiv \exists V_P. (\psi(N-1) \wedge D(V_Q, V_P))$$

$$\psi(N-1) \equiv \forall i \in [0, N-1], b[i] = i + (N-1)^2$$

$$\psi'(N-1) \equiv \forall i \in [0, N-1], b'[i] = i + N^2$$

Proving \exists Quantified Post-conditions

```
assume(true) // $\varphi(N)$ 
```

```
1. void imax(int A[], int N) { //PN
2.   int max = A[0];
3.
4.   for (int i=0; i<N; i++) {
5.     if(max < A[i]) {
6.       max = A[i];
7.     }
8.   }
9. }
```

```
assert( $\exists x \in [0, N), A[x]=max$ ) // $\psi(N)$ 
```

```
assume(true) // $\varphi'(N-1)$ 
```

```
1. void imax'(int A'[], int N) { //QN-1
2.   int max' = A[0];
3.
4.   for (int i=0; i<N-1; i++) {
5.     if(max' < A'[i]) {
6.       max' = A'[i];
7.     }
8.   }
```

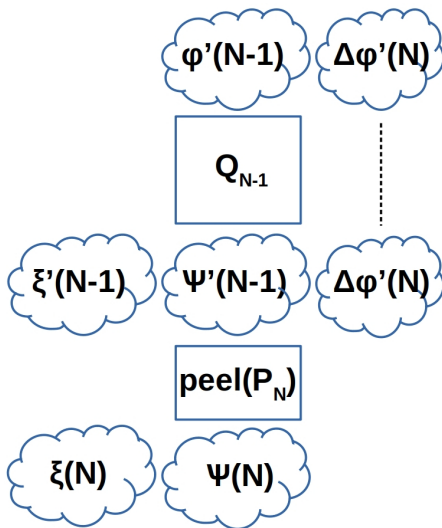
```
max'=max  $\wedge \forall x \in [0, N-1), A'[x]=A[x]$  //D(V', V)
```

```
assume( $\exists x \in [0, N-1), A'[x]=max'$ ) // $\psi'(N-1)$ 
```

```
9. if(max' < A'[N-1]) { // peel(PN)
10.  max' = A'[N-1];
11. }
```

```
assert( $\exists x \in [0, N), A'[x]=max'$ ) // $\psi(N)$ 
```

Strengthening Pre, Post using $\xi'(N-1)$, $\xi(N)$



Find ξ s.t. $\exists V_P. (\xi(N-1) \wedge D(V_Q, V_P)) \Rightarrow \xi'(N-1)$

Soundness Guarantee

Theorem

Suppose $\exists M > 0$ s.t.

- 1) $\{\varphi(N)\} P_N \{\psi(N) \wedge \xi(N)\}$ holds for $1 \leq N \leq M$, for some $M > 0$
- 2) $\xi(N) \wedge D(V_Q, V_P) \Rightarrow \xi'(N)$ holds for all $N > 0$
- 3) $\{\xi'(N-1) \wedge \Delta\varphi'(N) \wedge \psi'(N-1)\} \text{peel}(P_N) \{\xi(N) \wedge \psi(N)\}$ holds for all $N \geq M$

$\{\varphi(N)\} P_N \{\psi(N)\}$ holds for all $N > 0$

Benchmarking

- Developed a prototype tool [Diffy](#)
- Experiments on 303 benchmarks adapted from SV-COMP Arrays++
- Performance compared with the following tools:
 - ★ [Vajra](#) v1.0 - Chakraborty et. al.'2017
 - ★ [VeriAbs](#) v1.4.1-12 - Afzal et. al.'2020
 - ★ [VIAP](#) v1.1 - Rajkhowa & Lin'2019
- Intel i7-6500U CPU, 2.5 GHz, 16GB RAM, Ubuntu 18.04.5 LTS
- Time limit - 60s

Experimental Evaluation - Diffy

Program Category		Diffy			Vajra		VeriAbs		VIAP		
		S	U	TO	S	U	S	TO	S	U	TO
Safe C1	110	110	0	0	110	0	96	14	16	1	93
Safe C2	24	21	0	3	0	24	5	19	4	0	20
Safe C3	23	20	3	0	0	23	9	14	0	23	0
Total	157	151	3	3	110	47	110	47	20	24	113
Unsafe C1	99	98	1	0	98	1	84	15	98	0	1
Unsafe C2	24	24	0	0	17	7	19	5	22	0	2
Unsafe C3	23	20	3	0	0	23	22	1	0	23	0
Total	146	142	4	0	115	31	125	21	120	23	3

Table: Summary of the experimental results. S is successful result. U is inconclusive result. TO is timeout of 60s.

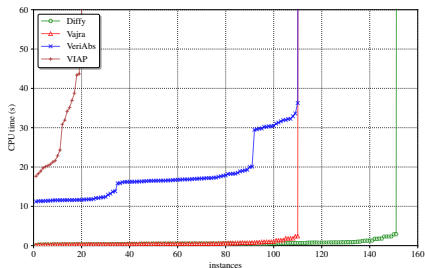
C1 - programs with standard array operations such as min, max, copy

C2 - branch conditions affected by N, modulo operators in programs

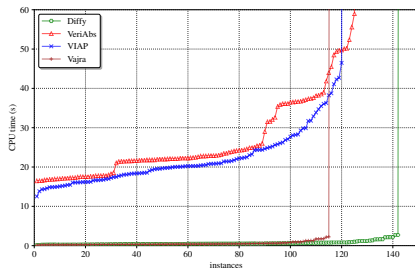
C3 - nested loop programs

Diffy out-performs winners of SV-COMP from Arrays sub-category

Performance Evaluation - Overall



(a)

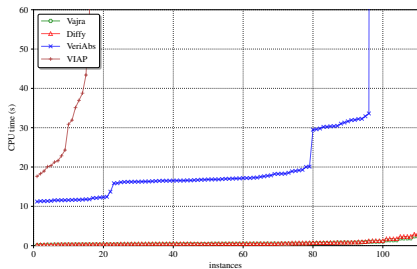


(b)

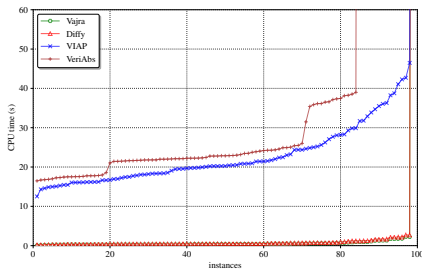
Figure: Cactus Plots (a) All Safe Benchmarks (b) All Unsafe Benchmarks

- ★ Diffy is $10\times$ faster on most benchmarks compared to VIAP, VeriAbs
- ★ Violations are reported by simple bmc when base case fails

Performance Evaluation - C1



(a)

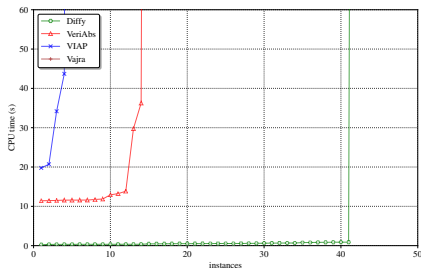


(b)

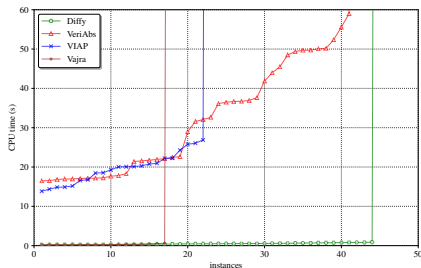
Figure: Cactus Plots (a) Safe C1 Benchmarks (b) Unsafe C1 Benchmarks

- ★ Diffy has comparable performance with Vajra
- ★ Diffy out-performs VeriAbs and VIAP

Performance Evaluation - C2 & C3



(a)



(b)

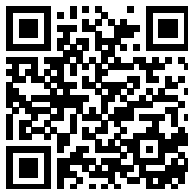
Figure: Cactus Plots (a) Safe C2 & C3 (b) Unsafe C2 & C3 Benchmarks

- ★ Nested loops are out-of-scope for Vajra and VIAP
- ★ VeriAbs verifies 3 programs from C2 and C3 that Diffy is unable to

Take Aways



Diffy Artifact



- Presented a novel, property driven and efficient technique that
 - ▶ performs simple transformations to enable full-program induction
 - ▶ computes *difference invariants* to aid the induction step
 - ★ often simpler than loop invariants
- Future work
 - ▶ Leverage translation validation for computing difference invariants
 - ▶ Expand scope beyond the currently supported programs

Thank you